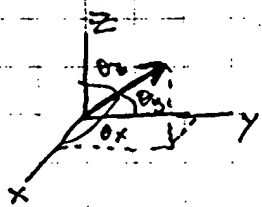


In this latter picture a par corresponds to ray ending exactly on last position removed from its original position.

- ① pick a direction vector
- ② calculate reflection off 3 sets of orthogonal planes
- ③ calculate travel distance between successive hits on a single set of planes
- ④ assign a loss/cm number

this looks amenable to an analytic treatment.

use direction cosines to parameterize ray direction

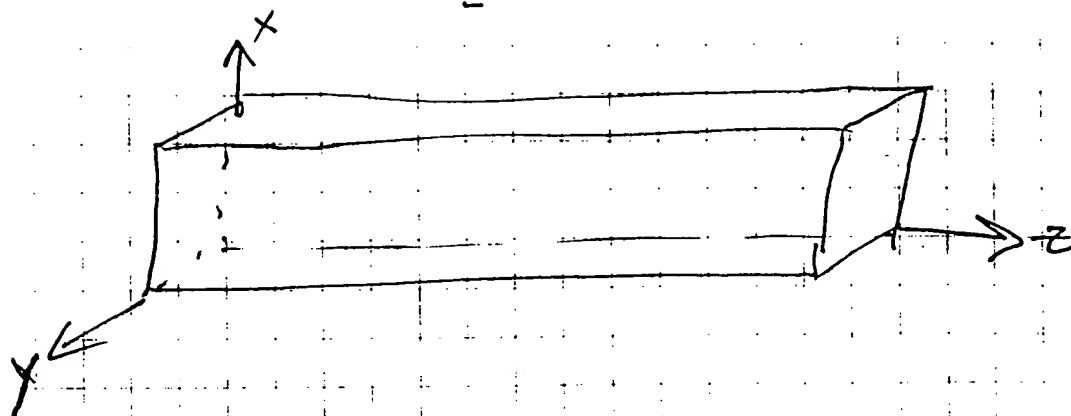


$$(\cos \theta_x, \cos \theta_y, \cos \theta_z) = \frac{(RND_1, RND_2, RND_3)}{\sqrt{RND_1^2 + RND_2^2 + RND_3^2}}$$

Let Δx , Δy , and Δz denote slab dimensions or plane spacing.

6-Sep-95

From this point of view it doesn't matter what position a ray is launched from, only its direction, because launch position has no impact on spacing between plane strikes.



Treat cants on surface perturbatively.

2 questions:

① How big do cants have to be to eliminate all parasites?

② For a rectangular slab, how close to slab index does cladding index have to be to eliminate all parasites.

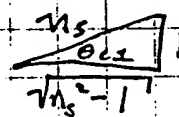
This question will be easiest to answer for a zero loss parasite

set z face incident angle equal to $\theta_{critz} = \sin^{-1}(\frac{1}{n_s})$ and the x face hit and y face hit also $= \theta_{critz} = \sin^{-1}(\frac{n_c}{n_s})$

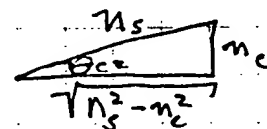
make this argument more rigorous!

Now work with direction cosines

$$\theta_{cz} = \sin^{-1}(\frac{1}{n_s})$$



$$\theta_{cy} = \sin^{-1}(\frac{n_c}{n_s})$$



$$\cos \theta_{cz} = \frac{\sqrt{n_s^2 - 1}}{n_s}$$

$$\cos \theta_{cy} = \frac{\sqrt{n_s^2 - n_c^2}}{n_s}$$

$$\cos^2 \theta_{cz} + 2 \cos^2 \theta_{cy} = 1$$

$$\frac{n_s^2 - 1}{n_s^2} + \frac{2(n_s^2 - n_c^2)}{n_s^2} = 1$$

$$n_s^2 - 1 + 2n_s^2 - 2n_c^2 = n_s^2$$

$$2(n_s^2 - n_c^2) = 1$$

$$n_s^2 - n_c^2 = \frac{1}{2}$$

when can this no longer be solved

$$n_c = \sqrt{n_s^2 - \frac{1}{2}}$$

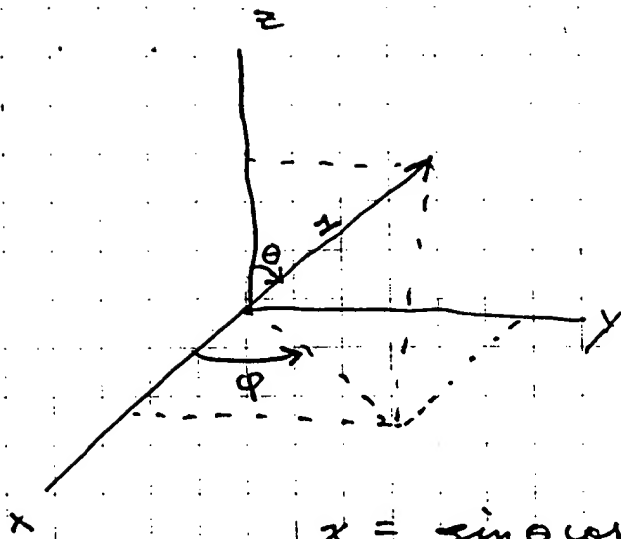
$$n_c = \sqrt{68.2^2 - \frac{1}{2}} = \underline{\underline{4.677}}$$

for $n_c > 4.677$ no zero loss
parasitics exist!

↳ this agrees with
code: slab ASE 01.XCL
prediction.

Question 2 will be easiest to answer numerically -
finding the angular width over which
a parasitic exists for given gain and
cladding indices.

7-sep-97



$$\begin{aligned} x &= \sin \theta \cos \phi = \cos \theta_x \\ y &= \sin \theta \sin \phi = \cos \theta_y \\ z &= \cos \theta = \cos \theta_z \end{aligned}$$

$$\begin{aligned} \theta_x &< \theta_{x \text{ crit}} \\ \theta_y &< \theta_{y \text{ crit}} \\ \theta_z &< \theta_{z \text{ crit}} \end{aligned}$$

> to avoid 0-loss parasites

$$\cos^2 \theta_x + \cos^2 \theta_y + \cos^2 \theta_z = 1$$

$$\begin{aligned} \cos \theta_x &> \cos \theta_{x \text{ crit}} \\ \cos \theta_y &> \cos \theta_{y \text{ crit}} \\ \cos \theta_z &> \cos \theta_{z \text{ crit}} \end{aligned}$$

> to avoid 0-loss parasites

$$\begin{aligned} \sin \theta_{x \text{ crit}} &= \frac{n_c}{n_s} \\ \sin \theta_{y \text{ crit}} &= \frac{n_c}{n_s} \\ \sin \theta_{z \text{ crit}} &= \frac{1}{n_s} \end{aligned}$$

$$\begin{aligned} \theta_{x \text{ crit}} &= \arcsin \left(\frac{n_c}{n_s} \right) \\ \theta_{y \text{ crit}} &= \arcsin \left(\frac{n_c}{n_s} \right) \\ \theta_{z \text{ crit}} &= \arcsin \left(\frac{1}{n_s} \right) \end{aligned}$$

$$\cos \theta_x > \frac{\sqrt{n_s^2 - n_c^2}}{n_s}$$

$$\cos \theta_y > \frac{\sqrt{n_s^2 - n_c^2}}{n_s}$$

$$\cos \theta_z > \frac{\sqrt{n_s^2 - 1}}{n_s}$$

> to avoid 0-loss parasites

$1 = \cos^2 \theta_x + \cos^2 \theta_y + \cos^2 \theta_z$ and this must be greater than

$$1 > \frac{n_c^2 - n_s^2}{n_s^2} + \frac{n_s^2 - n_c^2}{n_s^2} + \frac{n_s^2 - 1}{n_s^2}$$

to avoid 0-loss parasites

$$1 > \frac{3n_s^2 - 2n_c^2 - 1}{n_s^2}$$

n_s is slab index
 n_c is coating index

$$n_s^2 > 3n_s^2 - 2n_c^2 - 1$$

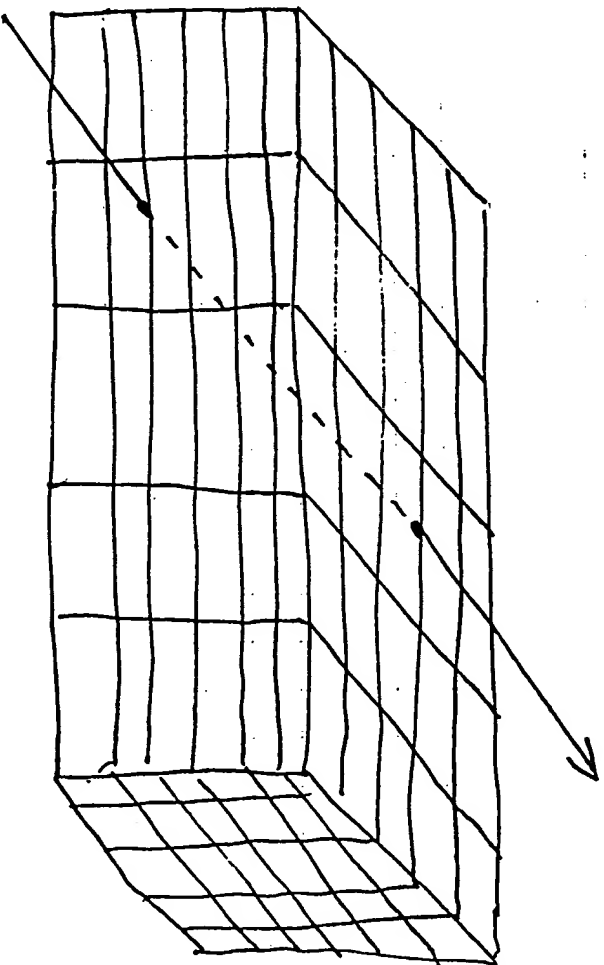
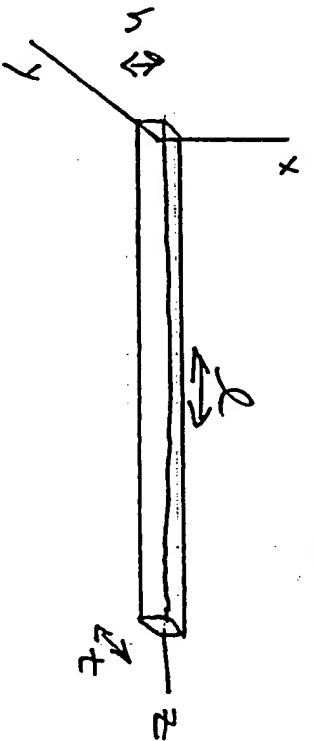
$$1 > 2(n_s^2 - n_c^2)$$

$$\frac{1}{2} > n_s^2 - n_c^2$$

$$n_c^2 > n_s^2 - \frac{1}{2}$$

$$n_c > \sqrt{n_s^2 - \frac{1}{2}} = \sqrt{1.82^2 - \frac{1}{2}} = 1.677$$

Using a method of images construction



fill space
with slabs
and use
images

- Define arbitrary ray direction using direction cosines $(\cos \theta_x, \cos \theta_y, \cos \theta_z)$
- Gain of ray in given in repeat/cum by $\frac{\sin(\text{Ref } x)}{\sin(\text{Ref } y)}$ and $\frac{\sin(\text{Ref } z)}{\sin(\text{Ref } x)}$

$$\rho = \frac{(h/\cos \theta_x)}{(t/\cos \theta_y)} \frac{\sin(\text{Ref } z)}{\sin(\text{Ref } x)} \alpha$$

where: $\text{Ref } i$ is the reflection coefficient for i-oriented planes
 α is slab specific gain loading

zero-loss parasitics correspond to those ray directions that are confined by TIR at all three sets of planes

$$\left. \begin{array}{l} \text{TIR} \\ \text{condition} \end{array} \right\} \begin{array}{l} \cos \theta_x < \cos \theta_{x\text{-crit}} = \frac{\sqrt{n_s^2 - n_c^2}}{n_s} \\ \cos \theta_y < \cos \theta_{y\text{-crit}} = \frac{\sqrt{n_s^2 - n_c^2}}{n_s} \\ \cos \theta_z < \cos \theta_{z\text{-crit}} = \frac{\sqrt{n_s^2 - 1}}{n_s} \end{array}$$

where:
 n_s = slab index
 n_c = coating index

• Since $1 = \cos^2 \theta_x + \cos^2 \theta_y + \cos^2 \theta_z$, zero loss parasitics exist when $1 < \frac{n_s^2 - n_c^2}{n_s^2} + \frac{n_s^2 - n_c^2}{n_s^2} + \frac{n_s^2 - 1}{n_s^2}$

or

$$n_c < \sqrt{n_s^2 - 1/2}$$

zero-loss parasitics can be completely suppressed by choosing a cladding with refractive index large enough

$$n_c > \sqrt{n_s^2 - 1}$$